## Mechanics Module 2 <br> Student Guide

## Concepts of this Module

- Vectors
- Relative Speeds
- Newton's First and Second Laws


## The Activities

## Course Activity 1

If we subtract a vector $\vec{A}$ from itself, there are at least two ways to write the result:

1. $\vec{A}-\vec{A}=\overrightarrow{0}$
2. $\vec{A}-\vec{A}=0$

The right hand side of the first form is a vector, while the right hand side of the second form is not. Which form is correct, 1 or 2 ? Why?

## Course Concepts

## Activity 2

Here are two position vectors, $\vec{A}$ and $\vec{B}$.


Vector $\vec{A}$ has a magnitude of 5.0 cm and makes an angle of 30 degrees with the vertical as shown. Vector $\vec{B}$ has a magnitude of 7.5 cm and makes an angle of 45 degrees with the vertical as shown.

Sketch the two vectors as shown onto the graph paper in your Lab Notebook. Note that sketch means a rough version: draftsman-like precision is not required. Leave at least 5 cm space around the sketch in all directions.
A. In the Notebook sketch and label the sum $\vec{A}+\vec{B}$
B. On the same sketch draw and label the sum $\vec{B}+\vec{A}$. Compare your result to Part A.
C. Sketch the two vectors into your Lab Notebook again. Leave at least 5 cm of space around the sketch in all directions. Draw and label the difference $\vec{A}-\vec{B}$.
D. On the same sketch as Part C, draw and label the difference $\vec{B}-\vec{A}$. Compare your results to Part C.

A simple little Flash animation illustrating addition of two vectors is available at:
http://faraday.physics.utoronto.ca/PVB/Harrison/Flash/Vectors/Add2Vectors.html
There is also an animation of subtracting two vectors at:
http://faraday.physics.utoronto.ca/PVB/Harrison/Flash/Vectors/Subtract2Vectors.html

## Course Activity 3

Here are three position vectors $\vec{A}, \vec{B}$ and $\vec{C}, \vec{A}$ and $\vec{B}$ are the same vectors as in Activity 2. Vector $\vec{C}$ has a magnitude of 7.0 cm at makes an angle of 30 degrees with the horizontal as shown.


Sketch the three vectors as shown into your Lab Notebook. Leave at least 5 cm space around the sketch in all directions.
A. In the Notebook sketch and label the sum $(\vec{A}+\vec{B})+\vec{C}$.
B. On the same sketch draw and label the sum $\vec{A}+(\vec{B}+\vec{C})$. Compare your result to Part A

A Flash animation illustrating the addition of three vectors is at:
http://faraday.physics.utoronto.ca/PVB/Harrison/Flash/Vectors/Add3Vectors.html

## Course Activity 4

A. Two vectors have different magnitudes. Can their sum be zero? Explain.
B. If one component of a vector is nonzero, can the vector have zero magnitude?

Explain.

## Courso Concepts <br> Activity 5

Assume that the speed of sound is exactly $344 \mathrm{~m} / \mathrm{s}$ relative to the air. Assume that the speed of light is exactly $3 \times 10^{8} \mathrm{~m} / \mathrm{s}$ relative to the observer.
A. If you are pursuing a sound wave at a speed of $99 \%$ of the speed of sound, what is the speed of the sound wave relative to you?
B. If you are moving through the air at $99 \%$ of the speed of sound in the opposite direction to the velocity of a sound wave, what is the speed of the sound wave relative to you?
C. If you are pursuing a light wave at $99 \%$ of the speed of sound, what is the speed of the light wave relative to you?
D. If you are pursuing a light wave at $99 \%$ of the speed of light, what is the speed of the light wave relative to you?


## Activity 6

We model Sponge Bob Square Pants as a simple sponge of width $w$, height $h$, and depth $d$. He is 6 cm wide, 12 cm high, and 4 cm deep.

It is raining. The raindrops are falling straight down at a constant speed of $9 \mathrm{~m} / \mathrm{s}$. Each raindrop has a diameter of 5 mm , and we can treat them as perfect spheres. There are 8000 raindrops per cubic meter.

A. Bob is stationary in the rainstorm. How many raindrops per second fall on the top of his head, i.e. the upper horizontal surface of the sponge? Do any raindrops strike his vertical surfaces?
B. Bob is now walking forward at $1.3 \mathrm{~m} / \mathrm{s}$. What is the velocity of the raindrops relative to Bob?
C. Now how many raindrops per second fall on the top of his head?
D. Bob is initially 50 m from a shelter. How many raindrops fall on the top of his head until he reaches the shelter?
E. How many raindrops per second strike his "face" i.e. the vertical surface of width $w$ and height $h$ ?
F. How many raindrops strike his face before he reaches the shelter?
G. Instead of walking, Bob runs for the shelter at $2.5 \mathrm{~m} / \mathrm{s}$. What is the velocity of the raindrops relative to Bob?
H. Now how many raindrops per second strike his "face" i.e. the vertical surface of width $w$ and height $h$ ?
I. Now how many raindrops strike his face before he reaches the shelter?
J. If it is raining, is it worth running for shelter instead of walking?

You can swim at a speed $v$ relative to the water. You are swimming across a river which flows at a speed $V$ relative to the shore. The river is straight and has a constant width.
A. If you wish to swim directly across the river, in what direction should you swim relative to the water in the river?
B. If you wish to get across the river as quickly as possible and don't care where you land on the opposite bank, in what direction should you swim relative to the water?

Joe is stationary on the ground, and sees an airplane moving to the right with a speed of


$$
\begin{gathered}
v=200 \mathrm{~m} / \mathrm{s} \\
a=5 \mathrm{~m} / \mathrm{s}^{2}
\end{gathered}
$$ $200 \mathrm{~m} / \mathrm{s}$ and accelerating at $5 \mathrm{~m} / \mathrm{s}^{2}$. Suzy is driving to the left at a constant $40 \mathrm{~m} / \mathrm{s}$ and Latoya is driving to the right at a constant 40 $\mathrm{m} / \mathrm{s}$.

A. Rank in order, from the largest to the smallest, the airplane's speed according to Suzy, Joe, and Latoya at the moment shown in the figure. Explain.
B. Rank in order, from the largest to the smallest, the magnitude of the airplane's acceleration according to Suzy, Joe, and Latoya. Explain.

## Preparing for Activities 9-11

Mechanics Module 1 included instructions for preparing for Activities 11 - 13 of that Module. You should use those instructions to prepare the apparatus for Activities 9 - 11 of this Module.

This Activity uses the Cart and Track that were introduced in Module 1. Use the thin blocks to raise the side of the Track closest to the wall a few millimeters. You will be finding the height for which the cart moves at constant speed, making the determination of whether the cart is moving at constant speed by eye; in Activity 11 some technology is used in place of your eyes to make the same determination.
A. Place the Cart on the Track near the end closest to the wall, place the supplied wooden block on the Cart, and give the Cart a very gentle push. Does it move at a constant speed down the track? If it is slowing down, raise the height a bit more. If it is speeding up, reduce the height. At what height does the Cart move at approximately constant speed? The playing cards are a good way to make small changes in the height.
B. When the Cart is moving at constant speed down the Track, sketch a Motion Diagram of its motion.
C. Treat the Cart plus the block on top of it as a single system. When the Cart is moving at constant speed down the Track, sketch a Free Body Diagram of all the forces acting on the system when it is about half-way down the Track.
D. How much can you vary the height of the track and not see any difference in the motion of the Cart? The playing cards are a good way to introduce small changes in the height.
E. Express your result from Part A as a single value. Include your result from Part D by adding $\mathrm{a} \pm$ uncertainty term to the value.
F. Place the wooden block in front of the Cart so the cart will push it down the Track. Now there will be more friction. Now what height must you raise the Track to have the Cart moving at approximately constant speed?
G. Again treat the Cart plus the wooden block as the system. Sketch a Free Body Diagram of all the forces acting on the system for Part F.
H. If you could completely eliminate the friction of the Cart and Track, what height would the end of the Track be raised for the Cart to move at constant speed?
I. Is it ever possible to completely eliminate friction?
J. Remove the wooden block but keep the Track at the same angle as Part F. Give the Cart a gentle push. Draw a Motion Diagram of its motion down the Track.
K. Draw a Free Body Diagram of all the forces acting on the Cart in Part J when it is about half-way down the Track. Compare to your Free Body Diagram of Part C.

For this Activity you will be using a computer-based laboratory system with an ultrasonic motion sensor and motion software. The motion sensor acts like a stupid bat when hooked up with a computer-based laboratory system. It sends out a series of sound pulses that are too high frequency to hear. These pulses reflect from objects in the vicinity of the motion sensor and some of the sound energy returns to the sensor. The computer is able to record the time it takes for the reflected sound waves to return to the sensor and then, by knowing the speed of sound in air, figure out how far away the reflecting object is.

There are a few points to be aware of when using the sensor:

1. The sensor cannot detect distances less than about 0.15 meters because it cannot record reflected pulses than come back too soon after they are sent.
2. The ultrasonic waves spread out in a cone of about $15^{\circ}$ as they travel. They will "see" the closest object. Be sure there is a clear path between the object you are tracking and the motion sensor.

For further details, see the manual on the detector and software at:
http://www.upscale.utoronto.ca/Practicals/Manuals/Equipment/MotionSensor/MotionSensor.pdf

Set the detector to collect about 40 samples per second. Set the switch on top of the sensor to the wide beam, which on some sensors is indicated by an icon of a person.

Use the system to take position-time data of one of your Team as he/she walks towards and away from the sensor. Try to glide as smoothly as possible at constant speed.

Loose clothing like bulky sweaters are good sound absorbers and may not be "seen" very well by the motion sensor.

The software will compute the average velocity and acceleration, just as you did by hand in Activity 8 of Module 1. Use the software to do those computations. Does the plot of average velocity look smooth? If not, why? What about the plot of average acceleration?

## Expl Activity 11

Mount the motion sensor on the end of the Track closest to the wall, and use the hardware and software to repeat Part D of Activity 9. Use the switch on top of the sensor to select the narrow beam, which on some sensors is indicated by an icon of a cart. Set the angle of the sensor to 0 so it is "looking" straight down the track.

Setup the sensor so it takes 5 samples per second. Do not try to measure distances less than about 0.15 m .
A. Does this do a better job than the estimates by eye that you did in Activity 10? Explain. In particular, is the $\pm$ uncertainty term using this method smaller than the result for Part E of Activity 9?
B. Save your distance-time data for one of your trials to the server by using the File tab of the Motion Sensor vi. Use a descriptive name for the file. Since this is Module 2, Activity 11, if you have raised the track by 3 mm the file name could be: M2A11_3mm. Write down the name of the file and the path in the lab book.
C. The datafile is a tab separated text file. Look at the file using either Excel or a text editor, but do not change the contents. For constant acceleration $a$, the distance $d$ depends on the initial distance $d_{0}$, initial speed $v_{0}$ according to:
$d=d_{0}+v_{0} t+\frac{1}{2} a t^{2}$

Use the PolynomialFit program, which is available on your computer's desktop, to fit the dataset to a second order polynomial (Powers 012 ). Is the acceleration of the Cart zero within uncertainties? How does using the Motion Sensor compare to you doing it by eye as in Activity 9?

In this Activity you will use a Force Sensor. When connected to appropriate software, this device measures forces exerted on it. The device uses a piezoelectric material, which generates a voltage proportional to the force exerted on it. Other uses of piezoelectrics include contact microphones, the motion sensor capabilities of the Sony Playstation 3 and Nintendo Wii controllers.

Pick out one of the Number 24 rubber bands as your standard rubber band. You may want to identify it by marking it with a pen or pencil. Loop the rubber band loose around your fingers as shown. Slowly separate your hands until the rubber band is not slack.


Now separate your hands by some further predetermined "standard" length that you choose. You can feel that the rubber band is exerting forces on both of your fingers. How do the magnitudes of these two forces compare?

Each member of your Team should do this simple little experiment.

A. When stretched by the standard length the rubber band is exerting a standard force on your fingers. Decide what name you wish to give to this standard force.
B. Now loop the rubber band around the hook on the Force Sensor that is mounted on the vertical rod and start the Force Sensor program on the computer. Push the Tare button on the Force Sensor to zero its reading. Stretch the rubber band by the standard length and determine the force in newtons corresponding to your standard force.
C. If you were to attach the rubber band to a Cart on the Track and pull the other end of the rubber band with your hand keeping it and stretched by your predetermined length, the Cart would accelerate. (This would take some physical dexterity to achieve.) What other ways can you think of to apply an equivalent standard force to the Cart?
D. How would you test to determine that these forces really are equivalent to your standard force?
E. Which of the methods you thought of in Part D do you think is the best one?

## Courso <br> Concopts <br> Activity 13

A. Loop the standard rubber band around your fingers and stretch it by your standard length to refresh your memory about what the standard force feels like. Now loop two rubber bands around your fingers and stretch them by your standard length. How does the force exerted on your fingers with two
 rubber bands compare to just one?
B. Repeat with three rubber bands.
C. Use the Force Sensor to check your feelings about the magnitudes of the forces.
D. Is there any difference between the forces exerted on the Force Sensor by a rubber band and an equal force exerted on it when you just hold the hook and pull? Explain.

In this Activity you will use a Fan Accessory. The fan accessory clamps to the collision cart and produces an approximately constant force upon it.

- Avoid a runaway Cart falling off the Track.

Level the Track and leave the Motion Sensor mounted on one end. Warm up the bearings of the wheels of the Cart by rolling it up and down the Track a few times. Set the fan angle at zero degrees.
A. Put 4 AA batteries in the Fan Accessory. Carefully clip the fan accessory to the top of the collision cart, avoiding putting too much pressure on the wheels of the cart. Place the Cart on the Track close to the Motion Sensor but at least 0.15 m away from it. You will want the direction of the air from the fan to blow towards the Motion Sensor. Turn the fan on and use the Motion Sensor to measure the acceleration of the Cart.
B. Sketch a motion diagram of the Cart.
C. Consider the Cart, motor, fan and the housing for the fan as the system under consideration. Sketch a Free Body Diagram of all the force acting on the system when the Cart was accelerating in Part A.
D. Use the Force Sensor or the spring-scale to measure the horizontal force acting on the system by the fan when it is not moving. You may wish to loop a length of
string over the small metal pin in the cart in order to attach the Force Sensor or spring-scale. Is this the force acting on the system when it is moving?
E. Repeat Parts A - D with two batteries swapped out for aluminum dummies. Note that the small screwdriver is supplied to help you pry out the batteries. This will halve the voltage provided to the fan motor, decreasing the fan speed and decreasing the force on the system. Put the real batteries back where the dummies were so that the mass of the system remains the same.
F. Sketch a graph of acceleration versus force, with the force on the horizontal axis. Be sure to include the origin on the graph. Although you only have two data points, what do you think the shape of the graph is for an arbitrary number of data points?
G. Is there a "free" third data point that you can include in your graph? Hint: what is the acceleration of the Fan Cart when the fan is off?
H. Sketch a straight line that "fits" the two data points. Should the line go through the origin?
I. How much can you vary the slope of the line and still more-or-less "fit" the data? Graphical estimation of slopes and their uncertainties is discussed in Section 14 Graphical Analysis of a document on uncertainty analysis at
http://www.upscale.utoronto.ca/PVB/Harrison/ErrorAnalysis/index.html
and also at:
http://www.upscale.utoronto.ca/PVB/Harrison/GUM/05_DataWithTwoVariables/05_DataWithTwoVariables.pdf
and also at:
http://www.upscale.utoronto.ca/PVB/Harrison/GUM/05 ExcelVersion/05_DataWithTwoVariables.pdf
Write down the relation between the force $F$ and acceleration $a$ as an equation including any necessary constants. Include as estimate of the uncertainty in those constants.

## Course Concopts <br> Activity 15

A key aspect of the scientific method is that often when a physical system has many variables we can keep all but two of the variables constant, and can investigate how those two variables relate to each other. In Activity 14 you varied the force applied to the Cart and saw how different forces cause different accelerations of the Cart. In this Activity you will apply the same constant force to the Cart but will vary its mass.
A. Measure the mass of the Fan Accessory, the Cart, and the two available masses.
B. The Fan Accessory snaps on top of the cart. In addition, the black rectangular masses can be placed on the Cart. How many possible values of the total mass are possible with and without the rectangular masses?
C. Measure the acceleration three different values of the mass.
D. Sketch a graph of acceleration versus total mass, with the mass on the horizontal axis. What is the shape of the graph?
E. Sketch a graph of acceleration versus one over the mass, with one over the mass on the horizontal axis. Include the origin in the graph. Is this graph simpler than the one in Part D?
F. For the graph of Part E, draw a straight line that "fits" the data. Should the line go through the origin? Why?
G. Write down the relation between mass $m$ and acceleration $a$ including any necessary constants and their uncertainties.
H. Combine your result for Part G and Activity 14 Part H into a single equation involving $F$ in newtons, $m$ in kg , and $a$ in $\mathrm{m} / \mathrm{s}^{2}$ and any necessary constants. You may find the following useful:

- What is the value of constant you found in Part J of Activity 14 in terms of any physical parameters of the system?
- What is the value of the constant you found in Part G of this Activity in terms of any physical parameters of the system?
I. Repeat Part H when the force is expressed in the unit you chose for the standard force in Activity 12 Part A. Are there any constants required now? Explain.

For one-dimensional motion in the x direction, here are three ways to write Newton's $2^{\text {nd }}$ Law:

1. $F_{x}=m a_{x}$
2. $a_{x}=\frac{1}{m} F_{x}$
3. $m=\frac{F_{x}}{a_{x}}$

Although these three forms are mathematically equivalent, in terms of using mathematics as a language to describe the relation between forces, masses and acceleration they are not. Which form best describes the central idea of Newton's $2^{\text {nd }}$ Law? Explain.

Hint: when you write that some variable $y$ is a function of another variable $x$, such as:

$$
y=\mathrm{f}(x)
$$

one variable is called the independent variable and another is called the dependent variable. Which is which, and why is this terminology used?

This Guide was written in July 2007 by David M. Harrison, Dept. of Physics, Univ. of Toronto. Some parts are based on Priscilla W. Laws et al, Workshop Physics Activity Guide (John Wiley, 2004) Unit 5. The figures in Activities 12 and 13 are modified from Randall D. Knight, Physics for Scientists and Engineers (Pearson Addison-Wesley, 2004), Figure 4.16.

Revisions by David M. Harrison, Jason Harlow and Lilian Leung. Last revision: May 20, 2014.

