

Mechanics Module 3 Student Guide

Concepts of this Module

- Equilibrium
- Mass and weight
- Two dimensional motion
- Projectile motion
- Circular motion
- Tensions and Ropes

Preparation for Activity 2 of this Module

In each of the four elevators in the tower of the Physics building is mounted a spring scale with a mass hanging from it. If Activity 2 is assigned, before the Practical take a ride in one of the elevators and note what happens to the reading of the scale for the six cases listed in Activity 2 below.

For your convenience, a form for writing down some data from your observations is available at:

http://www.upscale.utoronto.ca/Practicals/Modules/Mechanics/Mech Module03/ElevatorDataForm.pdf

Fill out the form and bring it to your Practical. Failure to do this will cause a deduction of one mark from your Practical mark.

The Activities



A round table is supported by three legs. If you are going to push down on the top of the table to make it unstable, where is the best place to push? Explain.



As preparation for this Module you took a ride on one of the elevators in the tower, paying attention to the reading of the spring scale for six different cases:

- a) Starts from rest and starts moving to a higher floor.
- b) Is moving uniformly up.
- c) Approaches the higher floor and starts slowing down.
- d) Starts from rest and starts moving to a lower floor.
- e) Is moving uniformly down.
- f) Approaches the lower floor and starts slowing down.

For each of the six cases:

- A. Describe the readings of the scale.
- B. Sketch a Free Body Diagram of all the forces acting on the mass during the motion being investigated. Use the diagram to explain the reading of the scale.
- C. Choose the upward direction as positive and the downward direction as negative, and select one of your Team's data. Calculate the acceleration of the elevator when it is:
 - a. Initially stationary and begins moving up the tower.
 - b. Moving up and slows down as it approaches a floor where it will stop.
 - c. Initially stationary and begins moving down the tower.
 - d. Moving down and slows down as it approaches a floor where it will stop.
- D. For each of the values of Part C, calculate the error in the value.
- E. How do the readings of the scale compare to what you felt for each of the six cases?
- F. Suppose that instead of a single mass suspended from a spring scale, the apparatus consisted of a *pan balance* with two masses with equal values on the pans. What would be the motion of this balance for each of the six cases you investigated? Explain.





In Jules Verne's **From the Earth to the Moon** (1865) a huge cannon fires a projectile at the moon. Inside the projectile was furniture, three people and two dogs. The figure is from the original edition.

Verne reasoned that at least until the projectile got close to the Moon it would be in the Earth's gravitational field during its journey. Thus the people and dogs would experience normal gravity, and be able to, for example, sit on the chairs just as if the projectile were sitting on the Earth's surface.

One of the dogs died during the trip. They put the dog's



body out the hatch and into space. The next day the people looked out the porthole and saw that the dog's body was still floating just beside the projectile.

- A. Is there a contradiction between the inhabitants inside the projectile experiencing normal gravity and the dog's body outside the projectile not falling back to the Earth?
- B. If your answer to Part A is yes, where did Verne make his mistake? If your answer is no, explain.



A bucket of water has a one end of a spring soldered to the bottom, as shown. A cork is attached to the other end of the spring and is suspended motionless under the surface of the water. You are holding the bucket so that it is stationary

- A. Draw a Free Body diagram of all the forces acting on the cork.
- B. As Archimedes realized a long time ago, the upward "buoyant" force on the cork is equal to the weight of the water that the cork has displaced. Imagine an identical



bucket-spring-cork system is stationary on the surface of the Jupiter where the acceleration due to gravity is 2.65 times greater than on Earth. Compared to the bucket-spring-cork on Earth, is the cork closer to the surface of the water, closer to the bottom of the bucket, or in the same relative position?

- C. Imagine that you take the Earth bucket-spring-cork onto an elevator. The elevator starts accelerating upwards. While it is accelerating does the cork move closer to the surface of the water, closer to the bottom of the bucket, or stay in the same relative position?
- D. Imagine that you take the Earth bucket-spring-cork up on the roof of a tall building. Still holding the bucket you step off. While you are in free fall towards the ground, does the cork move closer to the surface of the water, closer to the bottom of the bucket, or stay in the same relative position?



Wilma, queen of the drag strip, is about to race her Corvette Z06. She is stationary on the track, waiting for the lights to go green so she can accelerate down the strip. For luck, she always has a pair of fuzzy dice of mass m hanging from the rear view mirror.

We will model the dice hanging from the rear view mirror with the supplied ball and string.

One of your Team should hold the string with the ball hanging down. This person then begins walking forward at a fairly high speed.

- A. Before the person started walking sketch a Free Body Diagram of all the forces acting on the ball.
- B. Initially the ball was at rest for all of you. Newton's First Law says that bodies at rest remain at rest until a force causes their state of motion to change. When the person holding the ball begins walking what does he/she see the ball do? Is this what Wilma would see the fuzzy dice do? Are these consistent with Newton's First Law? Explain.
- C. For those of you who were not holding the ball and string, what did you see the ball do when the person holding the string began walking? Is this consistent with Newton's First Law? Explain.
- D. Assume Wilma is accelerating at a constant rate *a*. For you, standing beside the track, the dice reach a steady state where they are not hanging straight down, but make an angle θ with the vertical as shown. Draw the Free Body diagram of all external forces acting on the dice.
- E. What is the angle θ ?



Wilma, queen of the drag strip, has taken the kids to the zoo in her SUV. They are going home, and the kids are sitting in the back seat while the SUV is stopped at a stop light. Wilma bought them a Helium-filled balloon, which they are holding by the string so it is not touching the roof of the SUV. The balloon "floats" in the air because of a buoyant force on it, which Archimedes realized long ago is equal to the weight of the displaced air. The windows of the car are all rolled up. The light turns green and Wilma accelerates the SUV, but certainly at a lower rate than when she races her 'vette at the drag strip. Describe the motion of the balloon as seen by the kids after the light turns green.



A "funnel cart" has a ball on top of a funnel. Inside the funnel is an apparatus that fires the ball straight up at a predetermined time. If the cart is stationary, when the ball is fired it goes straight up and then lands back in the funnel.





- A. The cart is moving to the right at constant speed. When the ball is fired, does it land in the funnel? If not where does it land? Why?
- B. Now the cart is being pulled to the right and is accelerating. When the ball is fired, does it land in the funnel? If not where does it land? Why?
- C. Now the cart is rolling down a frictionless inclined track. Assume that the track is longer than is shown in the figure. When the ball is fired, does it land in the funnel? If not where does it land? Why?



Force



In Module 2 Activity 14 you used a *Fan* Accessory with an extra amount of mass sitting on the cart, as shown. Assume, as you did in that Activity, that the friction of the wheels is negligible. The extra mass on the cart has a mass *m*, and the mass of the cart, fan, motor etc. is *M*. A total force \vec{F} is exerted on the system. As you showed, the acceleration *a* of the system is:

$$a = \frac{F}{M+m}$$

- A. If the extra mass was not firmly attached to the cart, and, instead, was sitting on a super-slippery surface on top of the cart, when you released the Cart what would have been the motion of the mass *m*?
- B. For this case what would have been the acceleration of the Cart?
- C. In the actual case, the mass *m* moves along with the Cart with the same acceleration. Sketch a Free Body Diagram of all the forces acting on the mass *m* for this case.

- D. What is the magnitude and direction of the horizontal force exerted on mass m? What is the cause of this force?
- E. Sketch a Free Body Diagram of all the forces acting on the mass *M*.
- F. From Part E calculate the acceleration of mass *M*. Is your value reasonable?



Whirl the supplied ball on a string in a horizontal circle, being careful not to hit anybody or thing with it. Try to maintain the ball at constant speed.

- A. What is the net *vertical* force on the ball?
- B. Sketch a Free Body Diagram of the forces acting on the ball for some point in its circular orbit. There is a common convention for indicating vectors that are going out of or into the page, illustrated to the right. It is like an arrow: out of the new it is moving towards us we see the tip, but when it is moving away from us we see the feathers at the other end of the arrow.
- C. What must be the direction of the ball's acceleration to keep it moving in a horizontal circle?
- D. From you Free Body Diagram determine the net force acting on the ball. Does this agree with Part C?
- E. To maintain the ball at constant speed you need to move your hand that is holding the string. Explain why this is so. What would be the necessary condition to maintain the ball in uniform circular motion without needing to move your hand?
- F. If you suddenly let go of the string, what will be the motion of the ball? If you actually do this, be sure that you know in what direction the ball will go so that you don't hit anybody or thing.



Tarzan is swinging back and forth on a vine. We will model his motion with the supplied ball and string, and will assume that air resistance is negligible.

Fix the upper length of the string to a fixed point. Hold the ball so the string makes an angle of about $45^\circ = \pi/4$ radians and release it from rest so that it swings back and forth.

A. Using the supplied graph paper, draw a Motion Diagram for when the ball is released until it reaches its maximum swing on the other side. Use a total of 11



dots, with the 1st dot for the moment he steps off the branch, the 6th dot for when the vine is vertical, and 11th dot to the next position where the instantaneous speed is zero.

- B. Imagine that the dots in the diagram of Part A were for Tarzan's motion every second. (In other words, pretend the entire swing from start to stop takes 10 seconds.) Now draw an expanded scale Motion Diagram on another sheet of graph paper for the first second after he steps off the branch. Use 11 dots, each representing his position every 0.1 seconds. Connect the dots with vectors which are proportional the average velocity vectors.
- C. Re-draw the velocity vectors from Part B from a common origin. What is the direction of Tarzan's acceleration when he just steps off the branch?
- D. Sketch a Free Body Diagram of all the forces acting on Tarzan when he just stepped off the branch. What is the direction of the total force acting on him?
- E. Draw an expanded scale Motion Diagram for Tarzan's motion from 0.5 seconds before he reaches the bottom of his swing to 0.5 seconds after, again using a total of 11 dots. Connect the dots with vectors pointing from one position to the next.
- F. Re-draw the velocity vectors from Part E from the same origin. What is the direction of Tarzan's acceleration at the moment that the vine is vertical?
- G. Sketch a Free Body Diagram of all the forces acting on Tarzan when he is at the bottom of his swing? What is the direction of the total force acting on him?



Whirl the supplied ball on a string in a *vertical* circle. Have the ball moving fast enough that the string remains taut at all times.

- A. Qualitatively how does the speed of the ball at the top of the circle compare to its speed at the bottom of the circle?
- B. Sketch a Motion Diagram of the motion of the ball.
- C. Your hand can feel the tension the string is exerting on it. How does this tension related to the force being exerted on the ball? Qualitatively how does the force exerted on the ball at the top of the circle compare to the force exerted on it at the bottom of the circle?
- D. Allow the speed of the ball the decrease until the string is no longer taut at some point near the top of the circle. Sketch a Motion Diagram of the motion of the ball after this point in its motion.



Suppose you were to hang masses of m = 0.5 kg from the Force Sensors with light strings in the configurations shown below.



Predict the readings of the Force Sensors for each of A - G.

Check your prediction by doing the measurements. The sensor tends to "drift" in time. Therefore, before *each* measurement you should:

- 1. Have zero force being exerted on the sensor.
- 2. Press the *Tare* button on the sensor.

The strings that you are supplied have loops at their ends, and the Force Sensors have circular plastic connections. You can attach a string to the plastic connection by making a loop in the string as shown:



For Part G, you must connect the string to two Force Sensors. To do this, unscrew the plastic connectors from the sensors, connect them with two loops in the strings, the screw the connectors back into the sensors.





In the figure professional wrestler *Randy "Macho Man" Savage* is suspending a 10 kg mass with a rope between his two hands. Is the strongest member of your team, or even the Macho Man, strong enough to keep a heavy mass stationary and the rope perfectly horizontal? Explain





A wooden rod is suspended by a string tied to one end; the other end of the string is tied to a fixed support. The other end of the rod is resting on a piece of Styrofoam that is floating on water. Which figure is closest to the equilibrium position of the system?



Explain your answer.

Your Instructors will demonstrate this system. Was your prediction correct?



In the figure the Track is at an angle θ with the horizontal. The Cart has a mass *M* approximately equal to 0.5 kg. It is connected to a hanging mass $m = 0.0500 \pm 0.0001$ kg by a massless string over a massless pulley.

- A. Use the balance to measure the mass *M* of the Cart.
- B. For some angle θ the masses are in equilibrium, i.e. if they are at rest they remain at rest and if they are moving at some speed the continue moving at that speed. Calculate the value of the angle θ . Express your result in radians.¹
- C. The end of the Track that has the pulley mounted on it can be moved up and down using the attached clamp and the vertical rod mounted to the table. You may find that to make changes in the angle of the Track it is easiest to adjust the position of the vertical rod. Make sure that the pulley is not in contact with the Track, so that it turns freely. Verify your prediction of Part B. The digital angle gauge is a good

10

¹ 1 radian = 57.2958°, or 2π radians = 360°, or π radians = 180°.

way to measure the angle of the Track. Here is how to use the gauge:

- 1. Turn on by pressing the ON/OFF button. A digital readout should appear. If this does not happen, consult your demonstrator or the Resource Centre.
- 2. Find a surface and orientation of the gauge so that the bubble in the tube on top is centered. Press the ABS/ZERO button to zero the gauge.
- 3. Place the gauge on the surface to be measured. Allow it to settle. The reading error is 0.1° .

Note: because the Track is supported at both ends it tends to "sag" a bit in the middle. You will want to place the level fairly close to the position of the Cart.



This Activity continues the setup of Activity 15 Part C.

- A. By how much can you change the angle θ of the Track and not see any visible deviation from equilibrium. Express your result from Part C of Activity 15 and this Part by expressing the angle for equilibrium as $\theta \pm \Delta \theta$, with both values in radians.
- B. Imagine you are going to use this apparatus as a silly way of measuring the mass M of the Cart by measuring θ . Recall that $m = 50.0 \pm 0.1$ g. What is the value and error of M determined this way? What is the dominant error in your measurements that has the greatest effect on your value of ΔM ? [Note that if $\theta < ~10^{\circ}$ you may use the small angle approximation $\sin \theta \approx \tan \theta \approx \theta$, valid when θ is measured in radians.]
- C. The string is not really massless. Can you think of an experimental procedure for which the mass of the string does not matter?

Note: please turn off the digital angle gauge when you are finished with it by holding down the ON/OFF button for a few seconds.



Attach one end of a string to the pig and the other end to the hook in the middle of the top bar. Pressing the "on" button on the side of the pig will get its wings moving. Now take the pig and give it a push in a circular direction in order for it to start performing uniform circular motion. After a few seconds, its motion should be stabilized. Then you may begin your measurements.

- A. Determine the period of the pig's motion by finding the average time needed to complete 10 revolutions, using the stopwatch provided. Three trials are sufficient.
- B. Using the tape measure provided please measure the radius of the circle the pig is traveling. Determine a way to measure the pig's angle relative to the vertical.
- C. Using data from part A and B calculate the centripetal acceleration of the pig.
- D. Draw a free-body diagram for the pig (disregard "engine force" and air resistance as they cancel out).
- E. Use your free-body diagram and Newton's second law in the vertical direction to find the tension in the string.
- F. Use your free-body diagram and the tension above to solve for the net force in the radial direction.
- G. Compare the net force in part F to ma_c (using a_c from part C) How should these two values compare? How do your two values compare?
- H. Briefly discuss why the "engine force" and air resistance did not enter into the calculations.

This Guide was written in July 2007 by David M. Harrison, Dept. of Physics, Univ. of Toronto. Some parts are based on Priscilla W. Laws et al., **Workshop Physics Activity Guide** (John Wiley, 2004), Unit 7. Christos Josephides and Andrew Zasowski participated in development of the Mechanics Modules 1 - 4, and wrote much of Activity 9 of this Module.

Revised by Jason Harlow, David M. Harrison and Andrew Meyertholen. Last revision by Andrew Meyertholen, October 25, 2013.